4.2 Failure Criteria

The determination of residual strength for uncracked structures is straightforward because the ultimate strength of the material is the residual strength. A crack in a structure causes a high stress concentration resulting in a reduced residual strength. When the load on the structure exceeds a certain limit, the crack will extend. The crack extension may become immediately unstable and the crack may propagate in a fast uncontrollable manner causing complete fracture of the component.

Figure 4.2.1 illustrates the results obtained from a series of tests conducted on a lug geometry containing a crack. The lug geometry shown in Figure 4.2.1a is a single-load-path structure. Figure 4.2.1b indicates that the cracks in each of the three tests extended abruptly at a critical level of load, which is noted to be a function of a crack length. The crack length-critical load level data shown in Figure 4.2.1b provide the basis for establishing the residual strength capability curve. The locus of critical load levels as a function of crack length is shown in Figure 4.2.1c, where the residual strength capability of the lug structure is shown to decrease with increasing crack length.

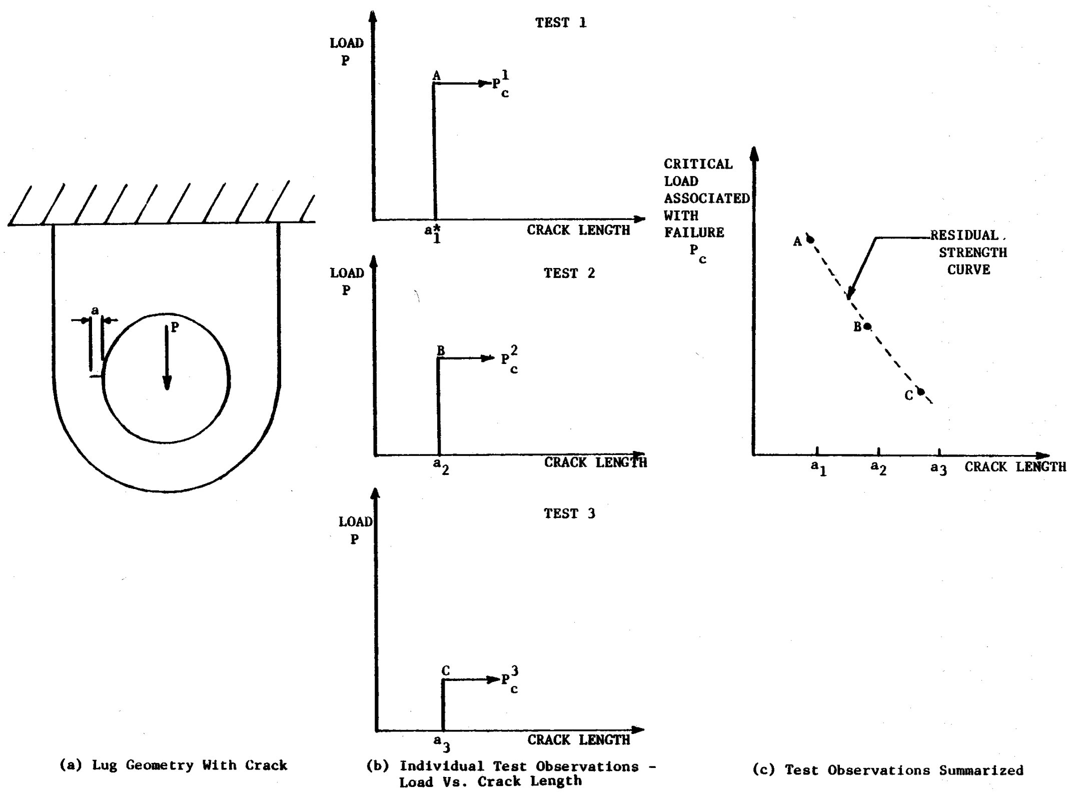


Figure 4.2.1. Description of Crack Geometry and Residual Strength Results

Considering the preceding in terms of applied stress (σ) rather than load gives the σ versus a and σ_c versus a_c plots as shown in Figure 4.2.2 a and b. Schematically, the plots exhibit the same abrupt fracture behavior as the curves presented in Figure 4.2.1. As also shown in Figure 4.2.2c and 4.2.2d, crack extension can first occur at a load level that is well below the fracture critical level. The point A′ corresponds to the start of slow and stable extension of the crack. The unstable rapid extension leading to total failure occurs at point A. This kind of behavior is observed typically in thin metal sheets or in tough materials. When different crack lengths are considered, the σ_c versus a_c plot will contain two distinct curves, as shown in Figure 4.2.2d. The curve A′B′C′ corresponds to the start of slow and stable crack extension and the curve ABC corresponds to failure.

In general, unstable crack propagation results in fracture of the component. Hence, unstable crack growth is what determines the residual strength. In order to estimate the residual strength of a structure, a thorough understanding of the crack growth behavior is needed. Also, the point at which the crack growth becomes unstable must be defined and this necessitates the need for a failure criterion. There are several criteria available; these criteria are tailored to represent the ability of a material to resist failure.

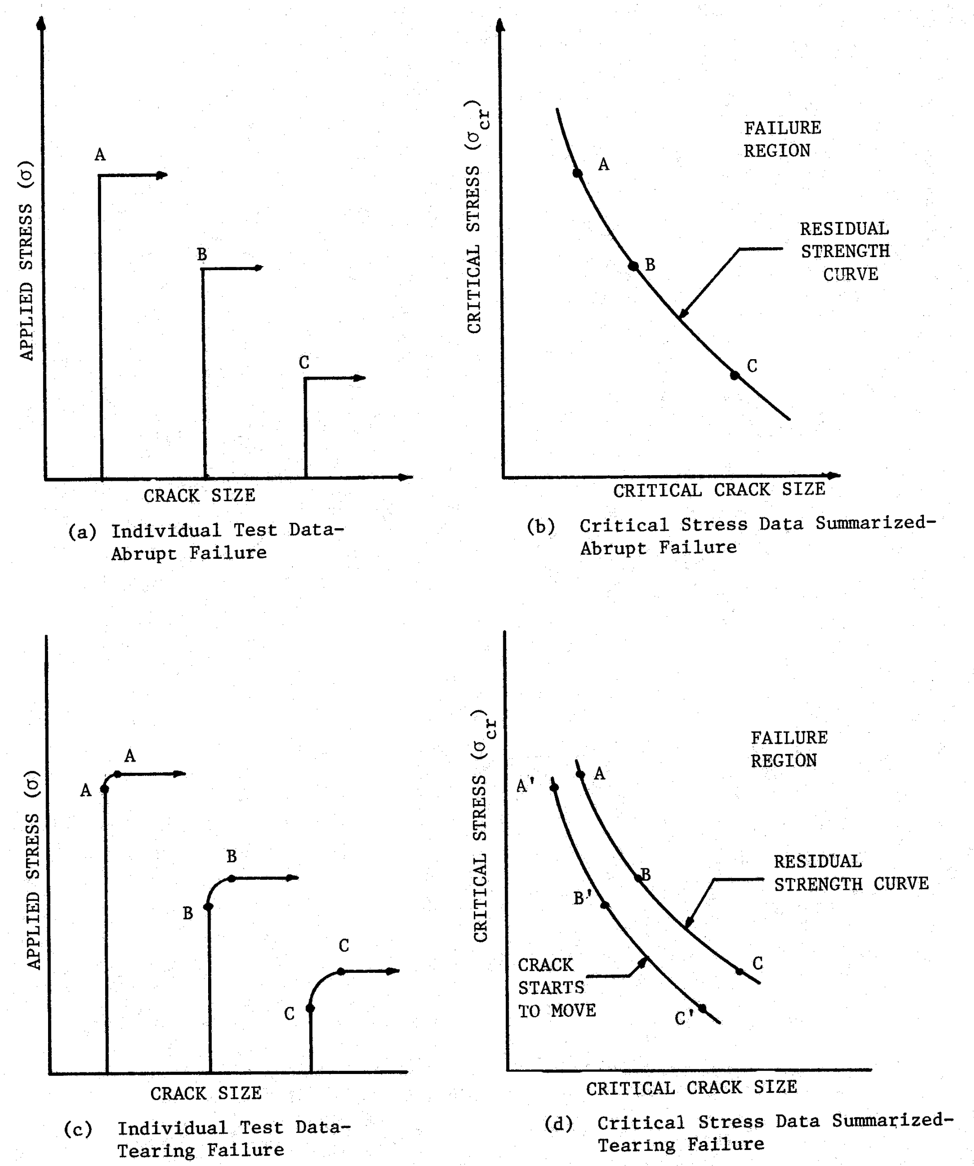


Figure 4.2.2. Fracture Data Described as a Function of Crack Length

4.2.1 Ultimate Strength

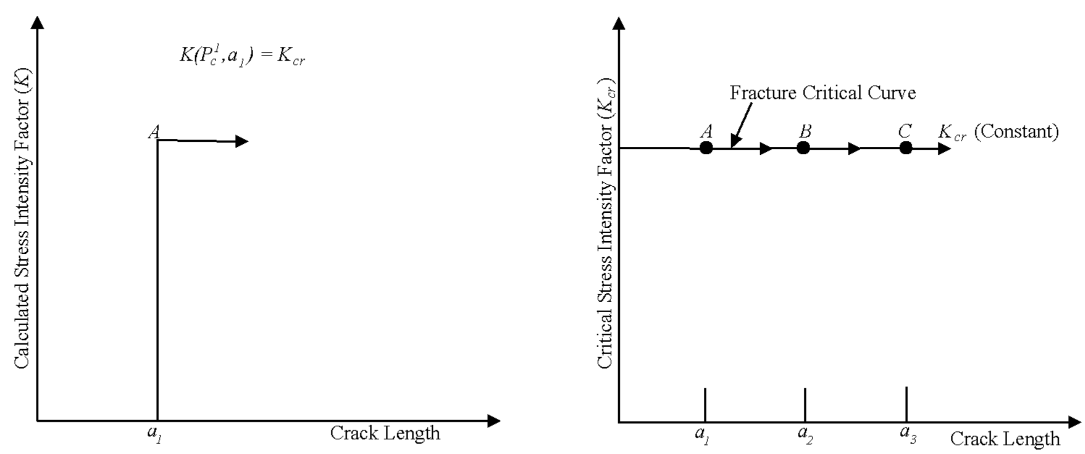
The simplest failure criterion assumes that failure occurs at the ultimate (or yield) strength of the material. Thus, the failure criterion becomes simply

$$\sigma_f = F_{tu} \tag{4.2.1}$$

where σ_f is the fracture stress and F_{tu} is the ultimate strength. This criterion is applicable primarily to uncracked structures and is included here for completeness. In past analyses of failure of built-up structure, the residual strength of stiffeners was based upon this criterion. When the main panel between the stiffeners fails due to catastrophic crack growth, the panel loads are transferred to the stringers (or stiffeners). The transferred loads may increase the stress level in the stringer so it is high enough to reach the value of σ_f, causing stiffener failure.

4.2.2 Fracture Toughness – Abrupt Fracture

In a cracked structure, as discussed in Section 2, the stress intensity factor (K) interrelates the local stresses in the region of the crack tip with crack geometry, structural geometry, and the level of load on the structure. When the applied load level increases, the K value also increases and reaches a critical value at which time the crack growth becomes unstable, as shown in Figure 4.2.3.



(a) Stress Intesity Factor Calculated as a Function of Load and Crack Length

(b) Critical Stress Intesity Factor Levels Described as a Function of Crack Length

Figure 4.2.3. The Fracture Mechanics Basis for Establishing Residual Strength

This critical level of K, which is independent of the crack length, is a material property called fracture toughness. The fracture toughness is a measure of the material's resistance to unstable cracking. Several test procedures are available to evaluate the fracture toughness. Also, various theoretical and numerical solution techniques are available, as discussed in Section 2, which can be used to estimate the (applied) stress intensity factor, K, for a given structure.

The failure criterion (Irwin's Criterion) states that abrupt fracture occurs when the crack-tip stress-intensity factor reaches or exceeds the fracture toughness of the material. The corresponding applied stress at failure is called the fracture strength. The failure criterion becomes simple

$$K \geq K_{cr} \tag{4.2.2}$$

where K_{cr} is the material's fracture toughness.

The critical K_{cr} for abrupt fracture mode is denoted as K_{Ic} for plane strain conditions and K_c for plane stress conditions; the conditions for plane stress or plane strain are determined by experiment. The test requirements necessary for generating K_{Ic} and K_c are discussed in Section 7.

The Damage Tolerant Design (Data) Handbook [Skinn, et al., 1994] contains a large quantity of fracture toughness data. Examples of the formats associated with individual test data for 7075 aluminum alloy are shown in Figures 4.2.4 and 4.2.5 for plane strain and plane stress fracture toughness values, respectively.

TABLE 8.9.2.1

1 of 36

ALUMINUM 7075 K_{Ic}

CONDITION	PRODUCT		TEST TEMP (°F)	SPEC OR	YIELD STR (Ksi)	SPECIMEN			CRACK LENGTH (in.) A	$2.5 \cdot (K_{Ic}/TYS)^2$ (in.)	K_{Ic}			DATE	REFER
	FORM	THICK (in.)				WIDTH (in.) W	THICK (in.) B	DESIGN			K_{Ic} (Ksi $\cdot$ $\sqrt{in.}$)	K_{Ic} MEAN	STAN DEV		
T6	Forging	0.50	R.T.	L-T	79.0	1.000	0.500	CT	0.534	0.23	24.20	24.3	0.1	1973	86213
		0.50			79.0	1.000	0.500	CT	0.523	0.24	24.40			1973	86213
T6	Forging	0.89	R.T.	T-L	67.2	0.500	0.249	NB	0.265	0.21	19.70	20.9	1.7	1973	86213
		0.89			70.0	0.500	0.249	NB	0.273	0.25	22.10			1973	86213
T6	Forging	0.50	R.T.	S-L	65.4	1.000	0.499	CT	0.493	0.17	17.00	16.8	0.4	1973	86213
		0.50			65.4	1.000	0.500	CT	0.510	0.16	16.70			1973	86213
		0.50			65.4	1.000	0.500	CT	0.496	0.16	16.40			1973	86213
		0.50			65.4	1.000	0.500	CT	0.505	0.17	17.20			1973	86213
T6	Forging	0.75	82	L-T	69.9	2.000	0.500	CT	1.025	0.44	29.20	---	---	1973	86213
T6	Forging	0.89	82	T-L	57.4	1.500	0.749	CT	0.785	0.32	20.40	19.4	1.7	1973	86213
		0.89			57.4	1.500	0.749	CT	0.762	0.32	20.40			1973	86218
		0.75			67.6	1.000	0.500	CT	0.511	0.17	17.50			1973	86213
T6	Forging	0.89	84	T-L	68.0	1.500	0.750	CT	0.792	0.24	21.20	20.6	0.8	1973	86213
		0.89			68.0	1.500	0.750	CT	0.798	0.22	20.00			1973	86213
T6	Extrusion	2.00	R.T.	T-L	72.0	1.500	0.750	CT	0.797	0.19	20.00	19.9	0.2	1973	86213
		2.00			73.6	1.500	0.749	CT	0.798	0.18	19.70			1973	86213
		2.00			73.6	1.500	0.748	CT	0.791	0.19	20.10			1973	86213
T6	Extrusion	2.00	R.T.	S-L	67.0	1.500	0.748	CT	0.791	0.19	18.50	18.5	0.2	1973	86213
		2.00			67.0	1.500	0.750	CT	0.798	0.19	18.30			1973	86213
		2.00			67.2	1.500	0.749	CT	0.808	0.19	18.70			1973	86213
T6	Forged Bar	---	R.T.	C-L	68.6	1.500	0.750	CT	0.750	0.20	19.60	19.5	0.2	1972	82879
		---			68.6	1.500	0.750	CT	0.750	0.20	19.30			1972	82879

Figure 4.2.4. Plane-Strain Fracture Toughness (K_{Ic}) Data for 7075 Aluminum in the Format of the Damage Tolerant Design (Data) Handbook [Skinn, et al., 1994]

TABLE 8.9.2.2

ALUMINUM 7075 K_C

CONDITION HEAT TREAT	PRODUCT FORM	PRODUCT THICK (in.)	TEST TEMP (°F)	SPEC OR	YIELD STR (Ksi)	SPECIMEN WIDTH (in.) W	SPECIMEN THICK (in.) B	CRACK LENGTH INIT (in.) $2a_o$	CRACK LENGTH FINAL (in.) $2a_f$	GROSS STRESS ONSET (Ksi) σ_o	GROSS STRESS MAX (Ksi) σ_{max}	K_{app} (Ksi$\sqrt{in}$)	K_{app} MEAN	K_{app} STAN DEV	K_c (Ksi$\sqrt{in}$)	K_c MEAN	K_c STAN DEV	DATE	REFER
BUCKLING OF CRACK EDGES RESTRAINED																			
T6	Sheet	0.10	R.T.	L-T	75.9	1.500	0.100	0.500	0.570	—	44.90	42.76*	—	—	46.72*	—	—	1962	62306
T6	Sheet	0.10	R.T.	L-T	75.9	3.500	0.100	0.500	0.760	—	53.70	48.20*	—	—	60.44*	—	—	1962	62306
		0.10			75.9	3.500	0.100	0.770	1.140	—	46.40	52.61			66.49*			1962	62306
T6	Sheet	0.10	R.T.	L-T	75.9	6.000	0.100	2.000	2.450	—	33.50	63.81	—	—	73.42	—	—	1962	62306
T6	Sheet	0.09	R.T.	L-T	75.9	12.000	0.090	1.040	1.460	—	46.60	59.84	64.6	3.2	71.22	71.9	2.8	1969	75599
		0.09			75.9	12.000	0.090	2.340	2.860	—	33.70	66.17			74.04			1969	75599
		0.09			75.9	12.000	0.090	1.060	1.560	—	44.80	58.09			70.87			1969	75599
		0.09			75.9	12.000	0.090	1.400	1.860	—	43.60	65.20			75.65			1969	75599
		0.09			75.9	12.000	0.090	3.880	4.720	15.30	25.20	66.55			76.00			1969	75599
		0.09			75.9	12.000	0.090	2.800	3.460	—	29.90	64.90			73.51			1969	75599
		0.09			75.9	12.000	0.090	1.100	1.440	—	44.00	58.14			66.77			1969	75599
		0.09			75.9	12.000	0.090	1.560	1.880	—	41.20	65.17			71.89			1969	75599
		0.09			75.9	12.000	0.090	4.500	5.460	11.40	22.50	65.60			75.82			1969	75599
		0.09			75.9	12.000	0.090	1.080	1.420	—	46.10	60.35			69.45			1969	75599
		0.09			75.9	12.000	0.090	1.820	1.920	—	40.00	68.61			70.58			1969	75599
		0.09			75.9	12.000	0.090	3.180	3.600	—	28.20	65.90			71.04			1969	75599
		0.09			75.9	12.000	0.090	3.060	3.580	—	28.20	64.43			70.80			1969	75599
		0.09			75.9	12.000	0.090	2.040	2.320	—	35.60	64.89			69.57			1969	75599
		0.09			75.9	12.000	0.090	2.620	3.500	—	29.60	61.88			73.29			1969	75599

* NOTE: NET SECTION STRESS EXCEEDS 80% OF YIELD STRENGTH. VALUE NOT INCLUDED IN MEAN OR STANDARD DEVIATION.

Figure 4.2.5. Plane-Stress Fracture Toughness (K_c) Data for 7075 Aluminum in the Format of the Damage Tolerant Design (Data) Handbook [Skinn, et al., 1994]

In general, a material's toughness depends on thickness, as shown in Figure 4.2.6. When the thickness is such that the crack tip plastic zone size is on the order of the plate thickness, the toughness reaches a maximum value, $K_{c(max)}$. With increasing thickness of the plate, the plastic zone size reduces and thus the toughness gradually decreases, from $K_{c(max)}$ to K_{Ic}. When the thickness is large enough that the crack tip deformation is not affected by the thickness, plane strain conditions prevail at the crack tip. The toughness in the plane strain regime is virtually independent of thickness. For increasing thickness, the toughness asymptotically approaches the plane strain fracture toughness, K_{Ic}.

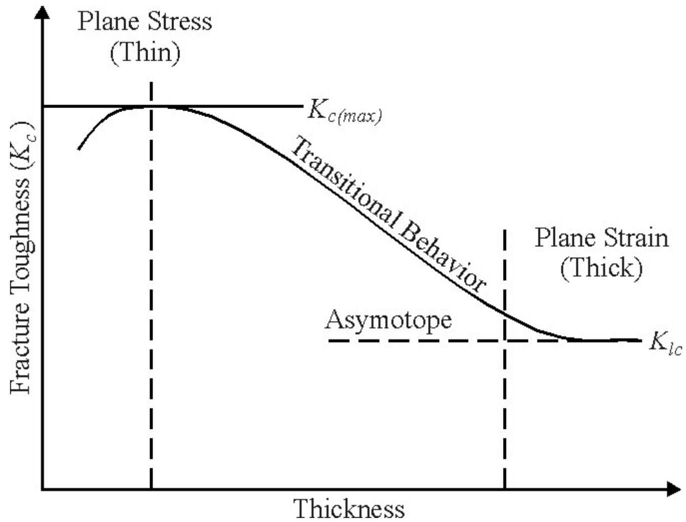


Figure 4.2.6. Fracture Toughness as a Function of Thickness

4.2.3 Crack Growth Resistance – Tearing Fracture

As illustrated in Figures 4.2.2c and d, when the crack extends by a tearing mode of fracture, which typically occurs in thin metal sheets or in tough materials, the crack extension is essentially slow and stable. The failure condition for tearing fractures depends on the crack growth resistance (K_R) behavior of the material and the applied stress-intensity factor K, which in turn depends on the crack and structural configurations. Figure 4.2.7 describes the observations that lead to the development of the crack growth resistance curve (K_R vs. Δa). Figure 4.2.7 a and b present the tearing behavior as a function of applied stress and the corresponding stress-intensity factor, respectively. Figure 4.2.7c presents the crack growth resistance curve that is a composite of the three stress-intensity factor curves shown in Figure 4.2.7b. Note that the composite was created by using the amount of physical crack movement observed in each case as the independent variable. As implied by the data points on the crack growth resistance curve in Figure 4.2.7c, the stress-intensity factor level associated with material failure is not necessarily constant.

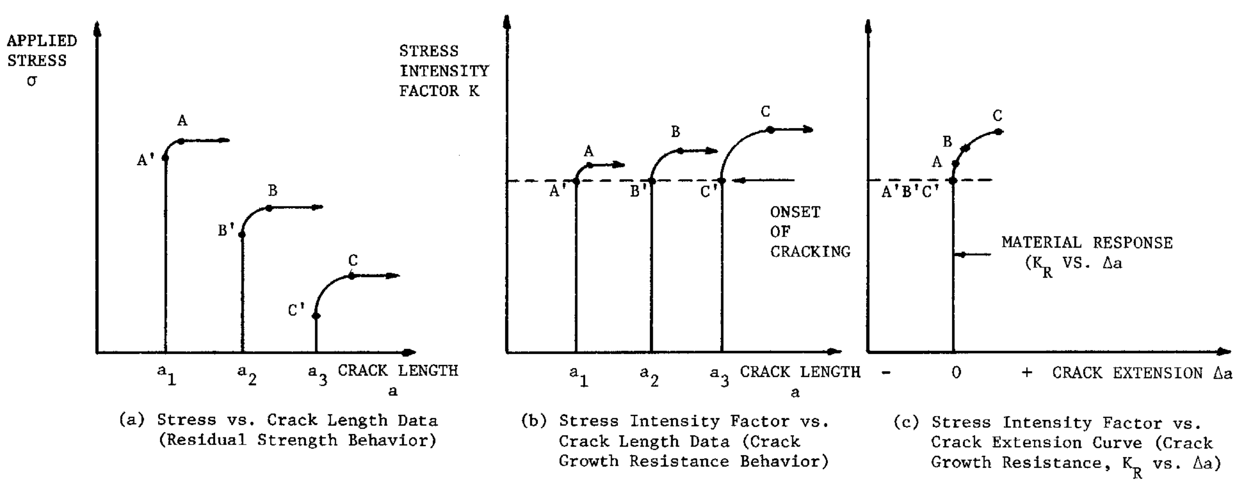


Figure 4.2.7. Schematic Illustration of Tearing Fracture Behavior and the Development of a Crack Growth Resistance Curve (R-Curve)

Shown in Figure 4.2.8 is a resistance curve for a 7475 aluminum alloy described as a function effective crack length [Margolis, et al., 1975]. The effective crack length is the sum of the physical crack length and the plastic zone size corresponding to the current crack length and loading conditions.

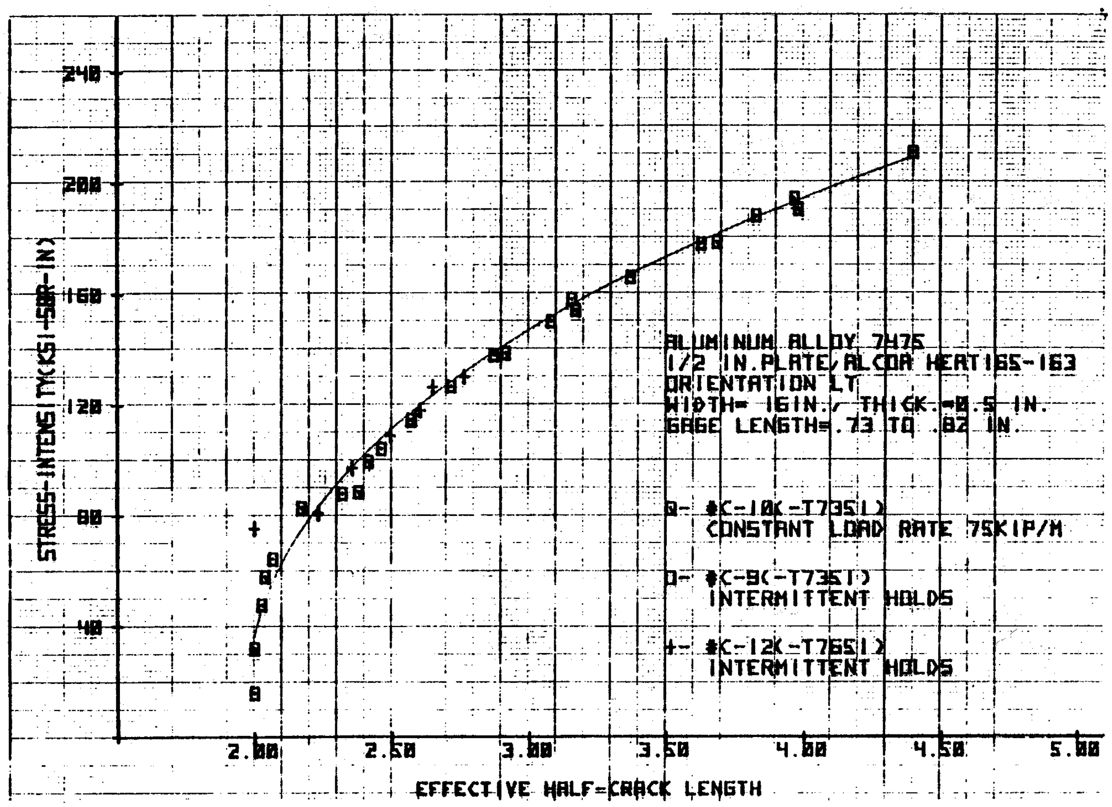


Figure 4.2.8. K_R Curve from 7475 Alloy, 16 Inch Wide Panels, 0.5 Inch Thick [Margolis, et al., 1975]

Indeed, while the shape of the resistance curve is basically independent of crack length or other geometrical effects, the fracture level is a function of crack length (see Figure 4.2.9). To account for this variation in fracture critical level, a two parameter failure criterion was required. However, before introducing the two parameter criteria that are used for more accurate estimates, approximate single parameter criteria for tearing failures are presented.

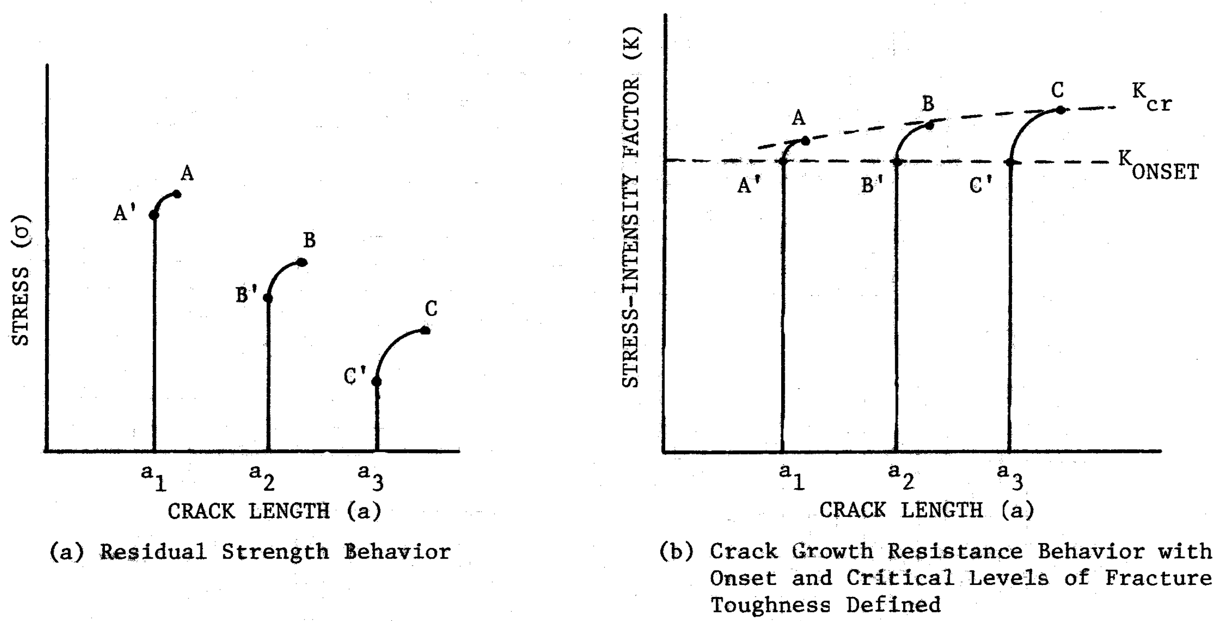


Figure 4.2.9. Schematic Illustration of Tearing Fracture Behavior Which Further Defines the Change in Critical Level of Fracture Toughness as a Function of Crack Length (also see Figure 4.2.7)

4.2.3.1 The Apparent Fracture Toughness Approach

Due to the complexity of the two parameter fracture criteria for tearing fracture behavior, engineers sometimes obtain preliminary estimates for the residual strength using a single parameter fracture toughness criterion. Figure 4.2.10 describes the stress-crack length levels associated with the onset of cracking ($K = K_{ONSET}$) and fast fracture ($K = K_{cr}$) conditions for a tearing material. Intermediate between the two curves established from material observations is a third curve referred to as the apparent fracture curve. The apparent fracture toughness (K_{APP}) is established from the same data employed to derive K_{ONSET} and K_{cr}. The calculation procedure uses the onset (or initial) crack length (a_i) and the final recorded stress level (σ_{cr}) for the tests conducted. Thus, K_{APP} represents a fracture toughness level bounded by the onset and fast fracture levels.

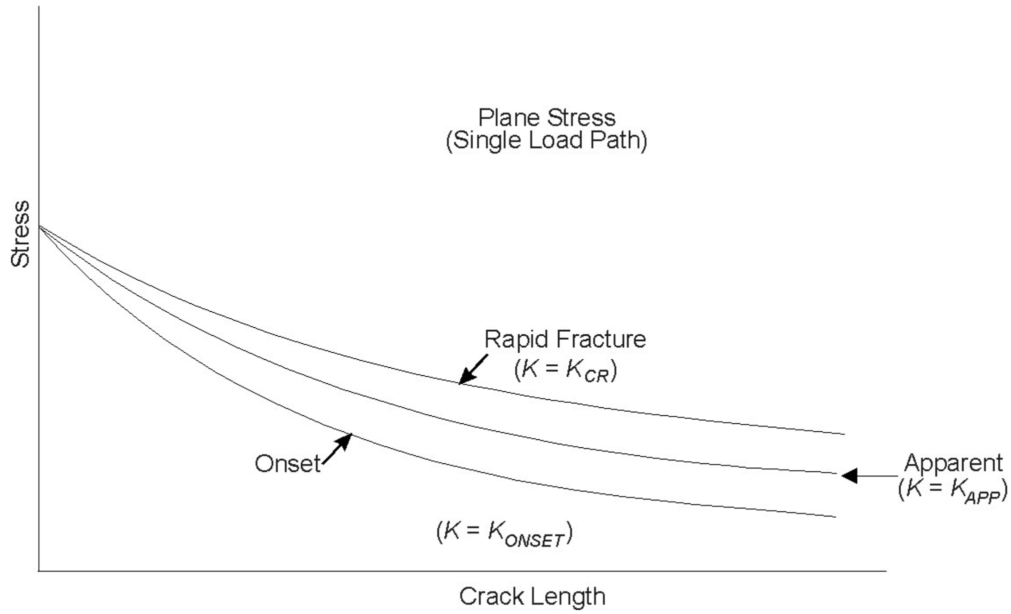


Figure 4.2.10. Description of the Three Fracture Toughness Criteria that are Utilized to Estimate Residual Strength Under Tearing Fracture Conditions

For lower bound estimates of the residual strength for fast fracture of a tearing material, one could equate the level of applied stress-intensity factor (K) to the apparent fracture toughness (K_{APP}), i.e., assume that fracture occurs when

$$K = K_{APP} \tag{4.2.3}$$

in order to determine the critical level of stress. Equation 4.2.3 is an abrupt failure criterion for a tearing fracture.

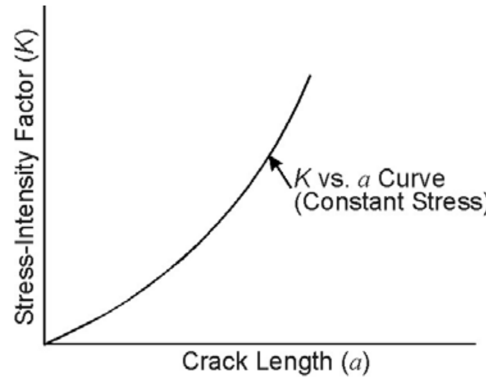
4.2.3.2 <u>The Resistance (*R*) Curve Approach</u>

If the crack tip plastic zone size is estimated to be on the order of the structural thickness but substantially smaller than other geometrical variables (crack length, ligament size, height, etc.), a linear elastic fracture mechanics analysis can still be sensibly used to predict the catastrophic cracking event. The failure criterion for tearing type fractures under these conditions states that fracture will occur when (1) the stress-intensity factor K reaches or exceeds the material's fracture resistance K_R and (2) the rate of change of K (with respect to crack length) reaches or exceeds the rate of change of K_R (with respect to crack length). Physically, the criterion means that at failure, the energy available to extend the crack equals or exceeds the material resistance to crack growth. The failure criterion becomes simply,

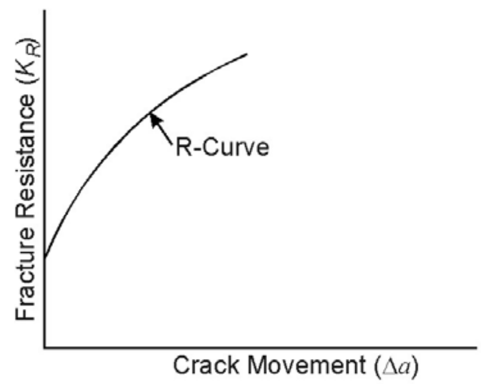
$$K \geq K_R; \quad \frac{\partial K}{\partial a} \geq \frac{\partial K_R}{\partial a} \tag{4.2.4}$$

The corresponding applied stress, σ_f, at this point is defined as the fracture stress that determines the residual strength of the cracked structure. The criterion presented in Equation 4.2.4 is noted to be a two-parameter criterion rather than the single parameter criteria that was presented in paragraph 4.2.3.1. To interpret the meaning of this criterion, first consider the structural parameters that are a function of the geometry and stress, i.e. K and $\frac{\partial K}{\partial a}$.

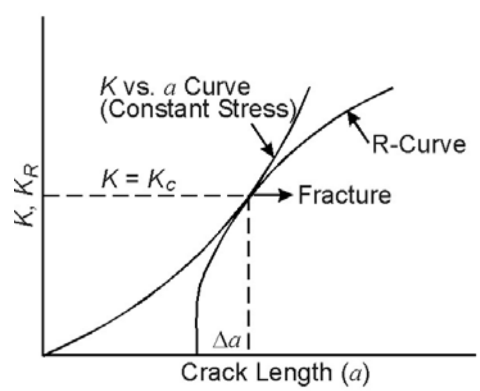
In general, the estimation of K involves the relationship $K = \sigma\beta\sqrt{\pi a}$ as given in Section 2; using this equation, the variation of K with respect to crack length (a) can be obtained for various values of stress (σ) as shown in Figure 4.2.11a. Shown in Figure 4.2.11b is the variation of K_R with respect to the crack extension (Δa) that was developed for the given material using the procedures outlined in Figure 4.2.7. Since this R-curve is assumed to be independent of the initial crack length, it can be superimposed on the plot of K versus a as shown in Figure 4.2.11c. The tangency point between the applied stress intensity factor curve (K vs. a) and the R-curve (K_R vs. Δa) determines the commencement of unstable crack propagation. In general, the accurate method of determining the tangency point involves the numerical solution based on the experimentally obtained R-curve. Using a least squares determined polynomial expression for R-curve and knowing an expression for K in terms of crack length, the common tangent point can be obtained by equating the functional values ($K = K_R$) and also the first derivatives with respect to the crack length $\left(\frac{dK}{da} = \frac{dK_R}{da}\right)$ of these two expressions.



(a) Driving Force



(b) Resistance to Crack Growth



(c) Establishment of Critical Conditions

Figure 4.2.11. Schematic Illustration of the Individual and Collective Parts of a K_R Fracture Analysis

The slow stable tear is dependent on a structural configuration in which the plastic zone at the crack tip is no longer negligible but not enormous. Krafft, et al. [1961], Srawley & Brown [1965], and McCabe [1973] explain the dependence of the *R*-curve on structural configuration as well as with test procedures used to evaluate the *R*-curve. See Section 7 for additional information on test procedures and the Damage Tolerant Design (Data) Handbook [Skinn, et al., 1994] for a summary of available data.

4.2.3.3 The *J*-Integral Resistance Curve Approach

The crack growth resistance curve (K_R) has shown good promise for materials where limited (small-scale) yielding occurs in front of the crack tip. Difficulties in estimating crack tip plasticity under large-scale yielding conditions, led Wilhem [1974] to an alternate failure criterion based on the *J*-integral [Rice, 1968]. For a basic introduction to the *J*-integral see Section 11.

Wilhem's *J*-integral criterion is similar to the K_R-curve criterion; it states that failure will occur when the following conditions are met:

$$\sqrt{J} \geq \sqrt{J_R}\,;\,\frac{d\sqrt{J}}{da} \geq \frac{\partial\sqrt{J_R}}{\partial a} \tag{4.2.5}$$

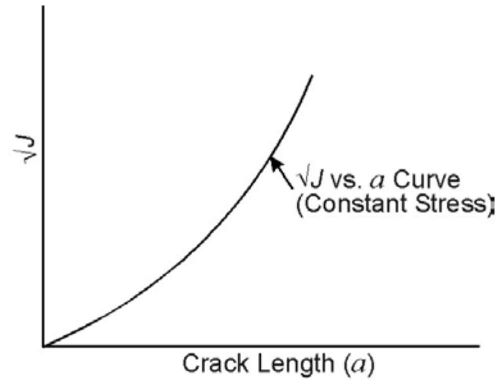
where J is the value of the applied *J*-integral and J_R is the value of the *J*-integral representing the material resistance to fracture. The applied stress (σ_f) corresponding to Equation 4.2.5 is defined as the fracture stress. Since the effect of large-scale yielding can be appropriately incorporated through a suitable elastic-plastic model in the estimation of *J*-integral, it becomes an effective parameter for predicting failure under plane stress conditions where the plastic zone size is significantly large.

The crack resistance curve for the tearing failure is now represented by $\sqrt{J_R}$ vs. Δa curve instead of K_R vs. Δa curve. The use of $\sqrt{J_R}$ rather than J_R is justified by the fact that $\sqrt{J}$ is directly related to the stress-intensity factor for elastic behavior through the equation

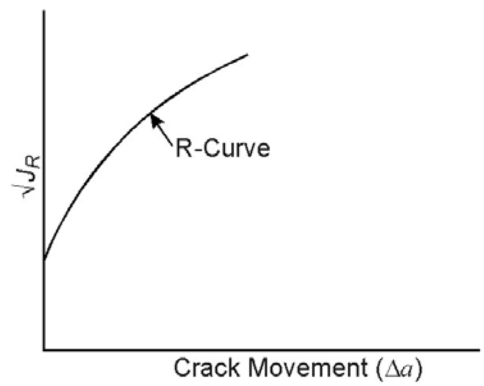
$$J = K^2/E' \tag{4.2.6}$$

where E' is the elastic modulus (E) for plane stress conditions and $E/(1-\nu^2)$ for plane strain conditions.

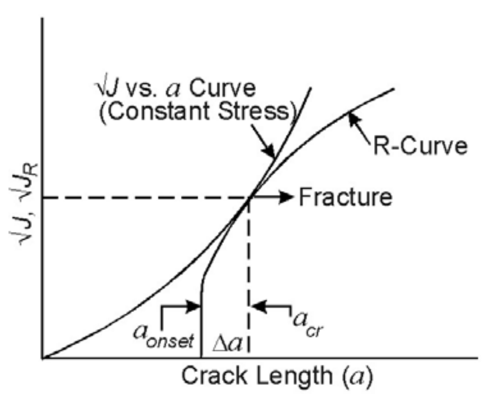
For different levels of applied load, the *J*-integral can be computed using finite element techniques for the structure of interest for a series of different crack sizes; the $\sqrt{J}$ versus crack length curve is illustrated in Figure 4.2.12a for a constant level of applied stress. It is noted that this curve will incorporate the influence of material properties (yield strength and strain hardening exponent) through the finite element analysis. In a manner similar to the stress-intensity factor type resistance curve, i.e. the K_R curve. The resistance curve based on $\sqrt{J_R}$ can be experimentally obtained [Griffis & Yoder, 1974; Verette & Wilhem, 1973]. A $\sqrt{J_R}$ versus crack movement (Δa) curve, i.e. the *J*-integral resistance curve, is schematically illustrated in Figure 4.2.12b. The failure criterion is also based on the tangency conditions between the $\sqrt{J}$ versus crack length curve and the $\sqrt{J_R}$ versus crack movement curve. To obtain this condition, the $\sqrt{J_R}$ vs. Δa curve can be superimposed on the plot of $\sqrt{J}$ vs. a curve such that at some crack length these two curves are tangent to each other as shown in Figure 4.2.12c. The corresponding crack length then defines the critical crack size of the structure for the fracture stress, σ_f.



(a) Driving Force



(b) Resistance to Crack Growth



(c) Establishment of Critical Conditions

Figure 4.2.12. Schematic Illustration of the Individual and Collective Parts of a J_R Fracture Analysis